

Show all working and note that late assignments will not be marked.

1. Solve the following equations (you should check that your solutions work).

a. $\log_2 x^2 = (\log_2 x)^2$ [2 marks]

b. $\frac{1}{e^{3x-1}} = 8$ [2 marks]

c. $\left(\frac{1}{3}\right)^{2x} = 81$ [2 marks]

d. A student solves $\log[(x+3)(x-8)] + \log\left(\frac{x+3}{x-8}\right) = 2$ by letting $x = 7$.

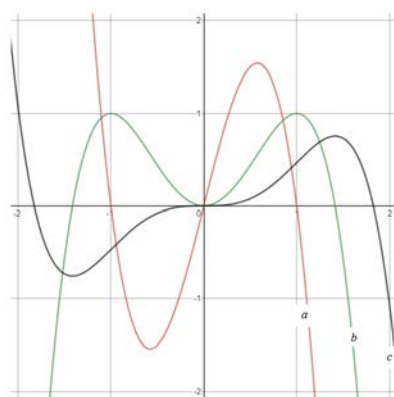
Using log rules she gets, $\log[(-10)(-10)] = \log(100) = 2$

i. Explain why the student's working is incorrect. [1 mark]

ii. Solve the equation $\log[(x+3)(x-8)] + \log\left(\frac{x+3}{x-8}\right) = 2$. Show your working.

[3 marks]

2. The figure shows graphs of f, f' and f'' . Identify each curve, and explain your choices. [6 marks]



3.

a. If $10^x = 2$ and $10^y = 3$ find 100^{2x-y} . [2 marks]

b. For $6e^{2x} - 7e^x - 20 = 0$

i. Show that if $y = e^x$ then $6e^{2x} - 7e^x - 20 = 0$ can be written as:

$6y^2 - 7y - 20 = 0$. [2 marks]

ii. Find the values of x . [2 marks]

4. It is estimated that the world population grew at an average annual rate of 1.37% in the twentieth century. The world population in 1900 was estimated to be around 1388 million (1.388 billion). The world population, N (in millions) in the twentieth century can be modelled by the equation

$$N(t) = 1510(1.0137)^t - 122, \text{ where } t \text{ is time in years.}$$

Assuming that the population grew in a constant rate,

- What was the estimated world population in 1905? [2 marks]
- In what year did the model predict the world's population to reach 3000 million (3 billion)? [3 marks]

5. The number of a certain type of bacteria in an area under controlled conditions is given by

$$B(t) = B_0 \cdot e^{\frac{-t}{2}}$$

(The initial number of bacteria is B_0 and t is measured in days.)

How long does it take for the initial number of bacteria to be reduced to 20%?

[3 marks]

6. Differentiate the following using the Chain Rule. [8 marks]

a. $f(x) = \sin^4(2x^3 - 1)$

b. $g(x) = \frac{6}{5x^3}$

c. $h(x) = 3\sqrt{1 - 2x^2}$

d. $y = \ln(1 + e^{-x})$

7. A giant lemon is thrown up in the air from an initial height of 2 metres. If the vertical height of the giant lemon relative to the ground, $h(t)$, after t seconds is given by the formula

$$h(t) = -16t^2 + 50t + 2,$$

find the height and the velocity of the giant lemon 3 seconds after it is thrown. Interpret your answers. [4 marks]

8.

- a. Write in sigma notation:

$$\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} \dots + \frac{7}{8} \quad [3 \text{ marks}]$$

- b. Calculate $\sum_{n=1}^5 \frac{2^n}{n!}$. Leave your answer in fraction form. [2 marks]

- c. $\sum_{i=3}^7 (100 - 2i^2) = 230$. Verify this by using $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$. [3 marks]

Total Marks: 50
